

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**BE - SEMESTER-IV (OLD) EXAMINATION – WINTER 2021**

Subject Code:140001

Date:31/12/2021

Subject Name:Mathematics-IV

Time:10:30 AM TO 01:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

**Q.1 (a)** Prove that  $\sinh^{-1} x = \log\{x + \sqrt{x^2 + 1}\}$ . **07**

**(b)** Solve the following system of equation by Gaussian elimination method **07**  
 with partial pivoting  $2x_1 + 2x_2 + x_3 = 6$ ;  $4x_1 + 2x_2 + 3x_3 = 4$  ;  
 $x_1 + x_2 + x_3 = 0$

**Q.2 (a)** Find the bilinear transformations which maps the points  $1, -1, \infty$  onto the **07**  
 points  $1 + i, 1 - i, 1$  respectively. Also, find its fixed points.

**(b)** Evaluate  $\int_C \frac{dz}{z^2 - 7z + 12}$ , where  $C$  is the circle  $|z| = 3.5$ . **07**

**OR**

**(b)** Evaluate  $\int_C \bar{z} dz$ , where  $C$  is along the sides of the triangle having **07**  
 vertices  $z = 0, 1, i$ .

**Q.3 (a)** Find and graph all sixth roots of unity. **07**

**(b)** Using the residue theorem, evaluate  $\int_C \frac{e^z + z}{z^3 - z} dz$ , where  $C: |z| = \frac{\pi}{2}$  **07**

**OR**

**Q.3 (a)** Find the analytic function  $f(z) = u + iv$ , if  $u - v = e^x(\cos y - \sin y)$ . **07**

**(b)** Expand  $\frac{1}{z^2 - 3z + 2}$  about  $z = 0$ , for the regions (i)  $1 < |z| < 2$  **07**  
 (ii)  $|z| > 2$ .

**Q.4 (a)** Apply fourth order Runge-kutta method to find  $y(0.2)$  given **07**

$$\frac{dy}{dx} = x + y, y(0) = 1. \text{ (Taking } h = 0.1)$$

**(b)** State Trapezoidal rule with  $n = 10$  and using it, evaluate  $\int_0^1 2e^x dx$  **07**

**OR**

**Q.4 (a)** Explain bisection method for solution of equation using this method find **07**  
 the approximate solution of  $x^3 - x + 1 = 0$  correct up to three decimal  
 points.

**(b)** Evaluate  $\int_{0.2}^{1.4} (\sin x - \log x + e^x) dx$  with  $h=0.2$  by simpson's  $1/3^{\text{rd}}$  and **07**  
 $3/8^{\text{th}}$  rule.

**Q.5 (a)** Find a real root of  $xe^x = 2$ , correct up three decimal places, by using Newton-Raphson method **07**

**(b)** Using Lagrange's interpolation formula obtain the value of  $x$  for  $y = 85$  **07**

X	2	5	8	14
Y	94.8	97.9	81.3	68.7

**OR**

**Q.5 (a)** Solve the following system of equations using Gauss-Seidel method: **07**  
 $5x + y - z = 10$  ;  $2x + 4y + z = 14$  ;  $x + y + 8z = 20$

**(b)** Compute  $f(9.2)$  from the following value Newton's divided difference formula. **07**

X	8.0	9.0	9.5	11
F(x)	2.079442	2.197225	2.251292	2.397895

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